Math 121: some more homework problems on differential equations:

(a) Chloe the canoeist is trying to become the first person to circumnavigate the globe in a canoe. She is making good progress, traveling at 2 m/s. Unfortunately, her paddle suddenly breaks, and she somehow forgot to bring a spare. From then on she is drifting, and she slows down at a rate proportional to the cube root of her velocity. After only one second, she has already slowed to 1 m/s, and she is despairing. Will she in fact stop, and if so, how long will it take? Perhaps irrationally, she clings to a faint hope that she will still make it all the way around the world, even though the circumference of the earth is about $3 \cdot 10^7$ meters. Will she in fact make it? If so, how long does it take?

(b) Chloe is back, and this time she has improved her canoe so that it drifts at a rate proportional to its velocity. Unfortunately, she got so wrapped up in the improvements to her canoe that she *again* forgot to bring a spare paddle. And wouldn't you know it, but her paddle breaks right when she's traveling at 2 m/s. After one second, she has again slowed to 1 m/s. Help her answer the same questions as in part (a).

(c) Chloe the indomitable canoeist has by this time attracted significant media attention, and along with that came a sponsorship from a small custom canoe manufacturer. Her new canoe drifts at a rate proportional to its velocity raised to the 4/3 power. However, she has no paddle sponsor, and somehow once again forgets to bring her own spare paddle. You can guess the rest – it unfolds just as before. Help her answer the same questions as in part (a).

(d) This time, Chloe is sponsored by CanoeKing™, the world's largest high-tech canoe manufacturer, and this sponsorship comes with a package of a dozen spare paddles. Her sleek new canoe drifts at a rate proportional to the cube of its velocity. Alas, all her spare paddles are eaten by a band of ravenous emus (their exponentially growing population has led to a severe famine). The same depressingly familiar sequence of events unfolds as in part (a). What happens this time?